Section 2.4 Notes: Real Zeros of Polynomial Functions

Long Division and the Division Algorithm

We have seen that factoring a polynomial reveals its zeros and much about its graph. Polynomial division gives us new and better ways to factor polynomials. First we observe that the division of polynomials closely resembles the division of integers:

Integer division	Polynomial division	
112	$1x^2 + 1x + 2$	← Quotient
$32\overline{)3587}$	$3x + 2\overline{)3x^3 + 5x^2 + 8x + 7}$	← Dividend
$\underline{32}$	$\underline{3x^3 + 2x^2}$	← Multiply: $1x^2 \cdot (3x + 2)$
387	$3x^2 + 8x + 7$	← Subtract
$\underline{32}$	$\underline{3x^2 + 2x}$	← Multiply: $1x \cdot (3x + 2)$
67	$6x + 7$	← Subtract
$\underline{64}$	$\underline{6x + 4}$	← Multiply: $2 \cdot (3x + 2)$
3	3	← Remainder

Division, whether integer or polynomial, involves a *dividend* divided by a *divisor* to obtain a *quotient* and a *remainder*. We can check and summarize our result with an equation of the form

$$(\text{Divisor})(\text{Quotient}) + \text{Remainder} = \text{Dividend}.$$

Synthetic Divison for polynomials:

A short cut method with the divisor is linear, and in the form of $x - k$

Use Synthetic Division to divide the following problems.

1) $(14x^2 + 2x^3 - 20x + 7) \div (x + 6)$

The dividend must be in standard form and fill in zeros for any missing terms

List the coefficients of the dividend (pay attention to signs!)

Use the opposite of the constant in divisor

Steps:

Bring down

Multiply

Add

Repeat

Use the coefficients of your answer to write the quotient.

The Quotient has degree ONE LESS than the dividend

Write a summary statement in fraction form:

2) $(3x^3 - 16x^2 - 72) \div (x - 6)$

The dividend must be in standard form and fill in zeros for any missing terms

List the coefficients of the dividend (pay attention to signs!)

Use the opposite of the constant in divisor

Steps:

Bring down

Multiply

Add

Repeat

Use the coefficients of your answer to write the quotient.

The Quotient has degree ONE LESS than the dividend

Write a summary statement in fraction form:

THEOREM Remainder Theorem

If a polynomial $f(x)$ is divided by $x - k$, then the remainder is $r = f(k)$.

3) Use the remainder theorem to find the remainder when f(x) is divided by x-k

$f(x) = x^3 - x^2 + 2x - 1; k = -3$

THEOREM Factor Theorem

A polynomial function $f(x)$ has a factor $x - k$ if and only if $f(k) = 0$.

4) Use the Factor theorem to determine whether the first polynomial is a factor of the second polynomial

$x - 2; x^3 + 3x - 4$

Fundamental Connections for Polynomial Functions

For a polynomial function f and a real number k, the following statements are equivalent:

1. $x = k$ is a solution (or root) of the equation $f(x) = 0$.
2. k is a zero of the function f.
3. k is an x-intercept of the graph of $y = f(x)$.
4. $x - k$ is a factor of $f(x)$.

5) Use synthetic division to show that x is a solution of the third degree polynomial equation, and use the result to factor the polynomial completely. Then, list all real solutions of the equation.

$$2x^3 - 15x^2 + 27x - 10 = 0, x = \frac{1}{2}$$

6) a) Verify the given factors of f(x) b) find the remaining factors of f(x)

c) use the results to write the complete factorization of f(x) d) list all zeros of f

Function $f(x) = x^4 - 4x^3 - 15x^2 + 58x - 40$ Factors $(x-5),(x+4)$

THEOREM Rational Zeros Theorem

Suppose f is a polynomial function of degree $n \geq 1$ of the form

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_0,$$

with every coefficient an integer and $a_0 \neq 0$. If $x = p/q$ is a rational zero of f, where p and q have no common integer factors other than ± 1, then

- p is an integer factor of the constant coefficient a_0, and
- q is an integer factor of the leading coefficient a_n.

Examples: